BENHA UNIVERSITY
FACULTY OF ENGINEERING AT SHOUBRA

Post-Graduate
ECE-601
Active Circuits

Lecture #4
Power Dividers and Directional Couplers

Instructor:
Dr. Ahmad El-Banna

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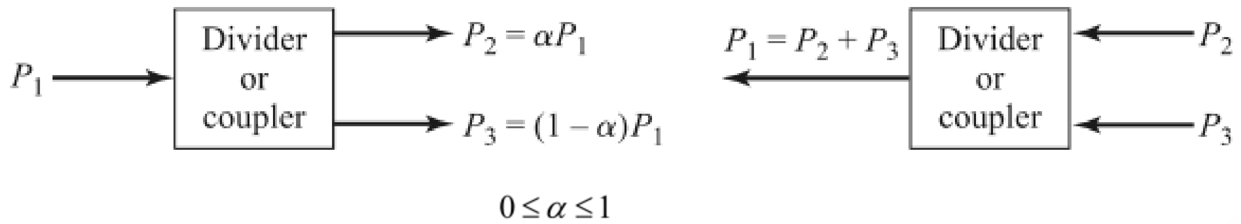
Agenda

- Introduction
- Basic Properties of Dividers and Couplers
- Types of Dividers and Couplers

INTRODUCTION

Introduction

- Power dividers and directional couplers are passive microwave components used for power division or power combining.
- In power division, an input signal is divided into two (or more) output signals of lesser power, while a power combiner accepts two or more input signals and combines them at an output port.
- The coupler or divider may have three ports, four ports, or more, and may be (ideally) lossless.
- Three-port networks take the form of T-junctions and other power dividers, while four-port networks take the form of directional couplers and hybrids.



Introduction..

- Power dividers usually provide in-phase output signals with an equal power division ratio (3 dB), but unequal power division ratios are also possible.
- Directional couplers can be designed for arbitrary power division, while hybrid junctions usually have equal power division. Hybrid junctions have either a 90◦ or a 180◦ phase shift between the output ports.
- Applications
  - Dividing (combining) a transmitter (receiver) signal to many antennas.
  - Separating forward and reverse propagating waves (can also use for a sort of matching).
  - Signal combining for a mixer.

BASIC PROPERTIES OF DIVIDERS AND COUPLERS

Three-Port Networks (T-Junctions)

- it's not possible to construct a three-port network that is:
  - lossless,
  - reciprocal, and
  - matched at all ports.
- A three-port network has an S matrix:

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- If the network is matched at every port, then $S_{11}=S_{22}=S_{33}=0$, and if the network is reciprocal, $S_{21}=S_{12}$, $S_{31}=S_{13}$, $S_{32}=S_{23}$.

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- If the network is lossless,

then $[S]$ is unitary.

$$[S]^t[S]^* = [U],$$

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{13}^* S_{23} = 0$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad S_{23}^* S_{12} = 0$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

→ at least two of the three S parameters must equal zero.
→ If this is the case, then not all of the equations can be satisfied.

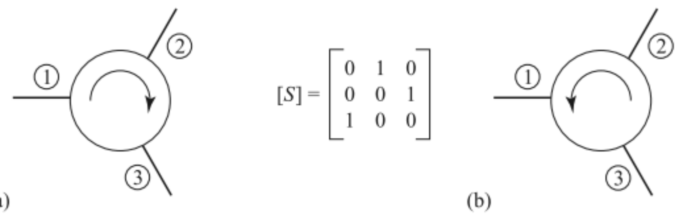
Three-Port Networks (T-Junctions)..

- Then a three-port network cannot be lossless, reciprocal, and matched at all ports. However, one can realize such a network if any of these three constraints is loosened.

Examples:

1. Nonreciprocal three-port: In this case, a lossless three-port that is matched at all ports can be realized. It is called a circulator.

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad [S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



Notice that $S_{ij} \neq S_{ji}$.

FIGURE 7.2 Two types of circulators and their scattering matrices. (a) Clockwise circulation. (b) Counterclockwise circulation. The phase references for the ports are arbitrary.

2. Match only two of the three ports. Assume ports 1 and 2 are matched.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}.$$

3. Lossy network. All ports can be simultaneously matched and the network reciprocal.

Four-Port Networks (Directional Couplers)

- Unlike three-ports, it is possible to make a lossless, matched, and reciprocal four-port network. These are called directional couplers.
- The S matrix of a reciprocal and matched four-port has the form:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- There are two commonly used realizations of directional couplers:
- 1-The Symmetrical Coupler. The S matrix for this device is

$$S_{12} = S_{34} = \alpha,\ S_{13} = \beta e^{j\theta},\ \text{and}\ S_{24} = \beta e^{j\phi}$$

α and β are real $\quad \alpha, \beta \in \mathbb{R}$ and $\alpha^2 + \beta^2 = 1$

θ and ϕ are phase constants

$$[S]=\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - It's called Quadrature (90°) Hybrid Coupler

$$\theta = \phi = \pi/2.$$

Four-Port Networks (Directional Couplers)

- 2. The Asymmetrical Coupler. The S matrix for this device is

$$\theta = 0, \phi = \pi.$$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

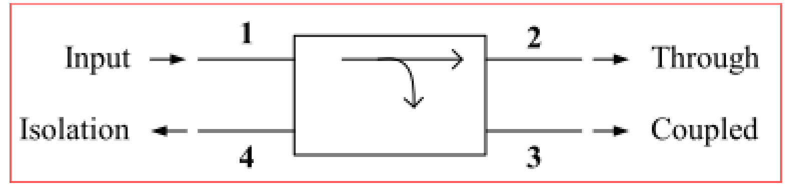
- The network is matched, reciprocal and lossless.
- It's called 180° Hybrid Coupler

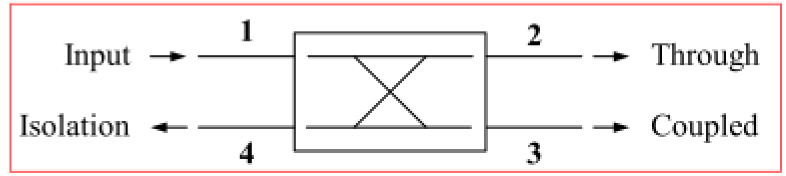
TYPES OF DIVIDERS AND COUPLERS

Types

- The T-Junction Power Divider
- The Wilkinson Power Divider
- Waveguide Directional Couplers
- The Quadrature (90◦) Hybrid
- Coupled Line Directional Couplers
- The Lange Coupler
- The 180◦ Hybrid

Directional Couplers





- The arrows indicate the assumed directions of time average power flow.
- The performance of directional couplers is characterized by the following values. For these definitions, port 1 is assumed the input, ports 2 and 3 the outputs, and port 4 is the isolated port.

$$\text{Coupling} = C = 10 \log \frac{P_1}{P_3} = -20 \log \beta \text{ dB},$$

$$\text{Directivity} = D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|} \text{ dB},$$

$$\text{Isolation} = I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}| \text{ dB},$$

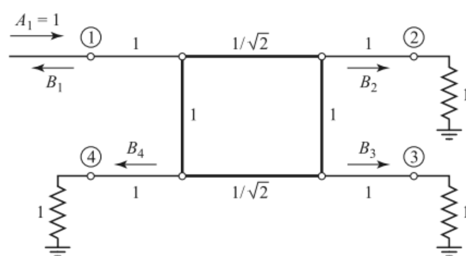
$$\text{Insertion loss} = L = 10 \log \frac{P_1}{P_2} = -20 \log |S_{12}| \text{ dB}.$$

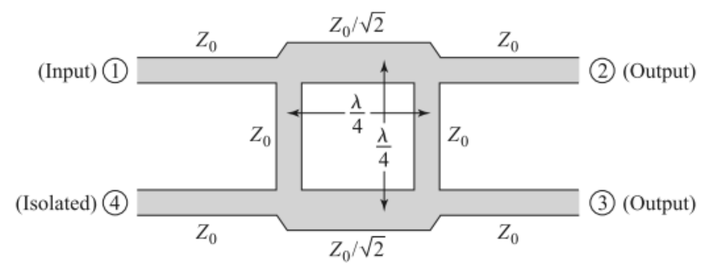
$$I = D + C \text{ dB}.$$

Hybrid Couplers

- Hybrid couplers are special cases of directional couplers, where the coupling factor is 3 dB, which implies that $\alpha = \beta = 1/\sqrt{2}$.
- There are two types of hybrids.
  - The **quadrature hybrid** has a 90◦ phase shift between ports 2 and 3 ($\theta = \phi = \pi/2$) when fed at port 1, and is an example of a *symmetric* coupler.

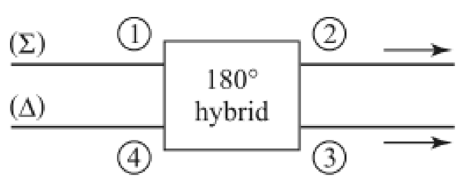
$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}.$$

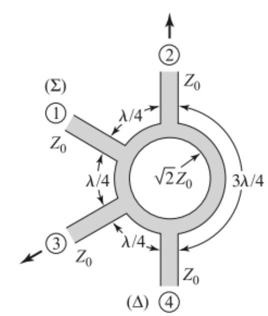




  - The **magic-T hybrid** and the **rat-race (ring) hybrid** have a 180◦ phase difference between ports 2 and 3 when fed at port 4, and are examples of an *antisymmetric* coupler.

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}.$$





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Examples

EXAMPLE 7.5 DESIGN AND PERFORMANCE OF A QUADRATURE HYBRID

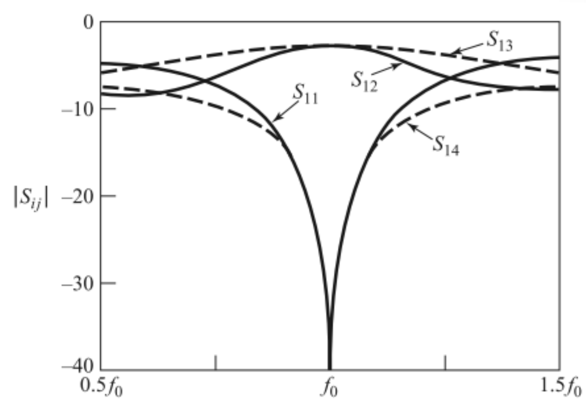
Design a 50 Ω branch-line quadrature hybrid junction, and plot the scattering parameter magnitudes from $0.5f_0$ to $1.5f_0$, where f_0 is the design frequency.

Solution

After the preceding analysis, the design of a quadrature hybrid is trivial. The lines are $\lambda/4$ at the design frequency f_0, and the branch-line impedances are

$$\frac{Z_0}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.4\ \Omega.$$

The calculated frequency response is plotted in Figure 7.25. Note that we obtain perfect 3 dB power division at ports 2 and 3, and perfect isolation and return loss at ports 4 and 1, respectively, at the design frequency f_0. All of these quantities, however, degrade quickly as the frequency departs from f_0. ■



EXAMPLE 7.9 DESIGN AND PERFORMANCE OF A RING HYBRID

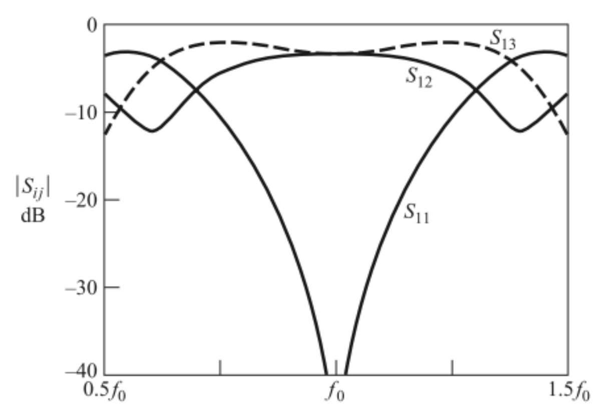
Design a 180° ring hybrid for a 50 Ω system impedance, and plot the magnitude of the scattering parameters (S_{1j}) from $0.5f_0$ to $1.5f_0$, where f_0 is the design frequency.

Solution

With reference to Figure 7.42a, the characteristic impedance of the ring transmission line is

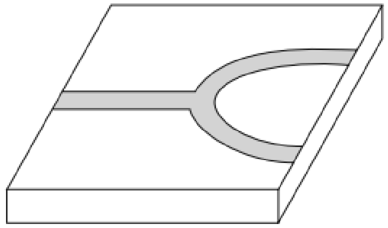
$$\sqrt{2}Z_0 = 70.7\ \Omega,$$

while the feedline impedances are 50 Ω. The scattering parameter magnitudes are plotted versus frequency in Figure 7.46. ■



THE T-JUNCTION POWER DIVIDER

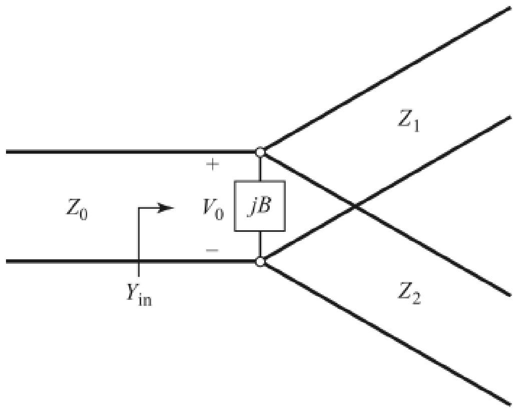
- The T-junction power divider is a simple three-port network that can be used for power division or power combining, and it can be implemented in virtually any type of transmission line medium.



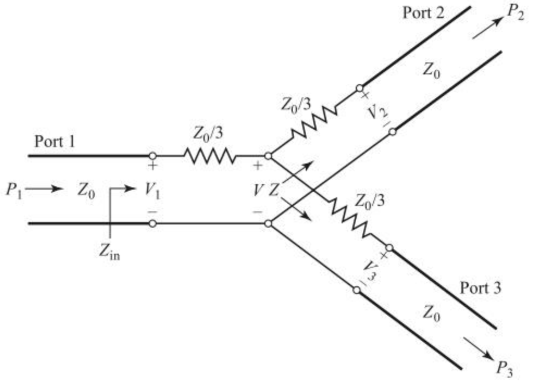
- Lossless Divider:

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}.$$

assume $B = 0$, $$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}.$$



- Resistive Divider:



Example

EXAMPLE 7.1 THE T-JUNCTION POWER DIVIDER

A lossless T-junction power divider has a source impedance of 50 Ω. Find the output characteristic impedances so that the output powers are in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports.

Solution

If the voltage at the junction is V_0, as shown in Figure 7.6, the input power to the matched divider is

$$P_{in} = \frac{1}{2}\frac{V_0^2}{Z_0},$$

while the output powers are

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{in},$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{in}.$$

These results yield the characteristic impedances as

$$Z_1 = 3Z_0 = 150\ \Omega,$$

$$Z_2 = \frac{3Z_0}{2} = 75\ \Omega.$$

The input impedance to the junction is

$$Z_{in} = 75||150 = 50\ \Omega,$$

so that the input is matched to the 50 Ω source.

Looking into the 150 Ω output line, we see an impedance of 50 || 75 = 30 Ω, while at the 75 Ω output line we see an impedance of 50 || 150 = 37.5 Ω. The reflection coefficients seen looking into these ports are

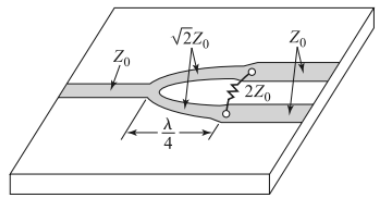
$$\Gamma_1 = \frac{30 - 150}{30 + 150} = -0.666,$$

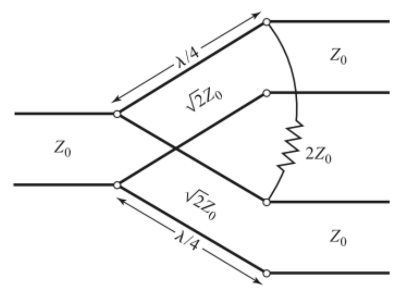
$$\Gamma_2 = \frac{37.5 - 75}{37.5 + 75} = -0.333.$$

■

THE WILKINSON POWER DIVIDER

- This is a popular power divider because it is easy to construct and has some extremely useful properties:
  - Matched at all ports,
  - Large isolation between output ports,
  - Reciprocal,
  - Lossless when output ports are matched.

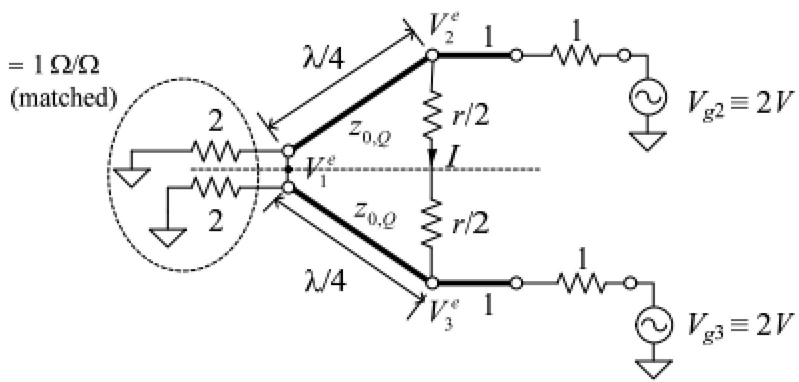




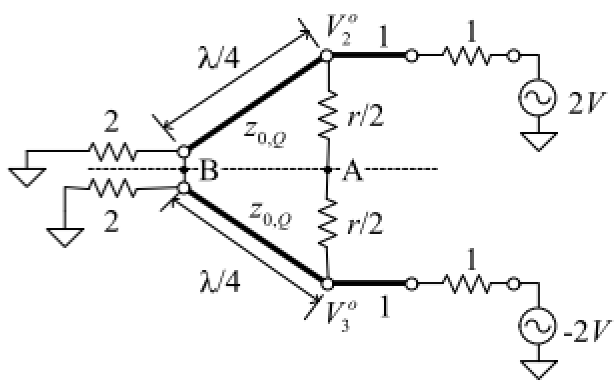
- There is much symmetry in this circuit that can be exploited to make the S parameter calculations easier.
- Specifically, we will excite this circuit in two very special configurations (symmetrically and anti-symmetrically), then add these two solutions for the total solution.
- This mathematical process is called an “even-odd mode analysis.” It is a technique used in many branches of science such as quantum mechanics, antenna analysis, etc.

even-odd mode analysis

- **Symmetric excitation (even mode):**



- **Anti-symmetric excitation (odd mode):**



$$S_{11} = 0$$

$$S_{33} = S_{22}.$$

$$S_{22} = 0 = S_{33}$$

$$S_{12} = -\frac{j}{\sqrt{2}} = S_{21}$$

$$S_{13} = S_{31} = -\frac{j}{\sqrt{2}}$$

$$S_{32} = \frac{V - V}{V + V} = 0 = S_{23}$$

Example

EXAMPLE 7.2 DESIGN AND PERFORMANCE OF A WILKINSON DIVIDER

Design an equal-split Wilkinson power divider for a 50 Ω system impedance at frequency f_0, and plot the return loss (S_{11}), insertion loss ($S_{21} = S_{31}$), and isolation ($S_{23} = S_{32}$) versus frequency from $0.5 f_0$ to $1.5 f_0$.

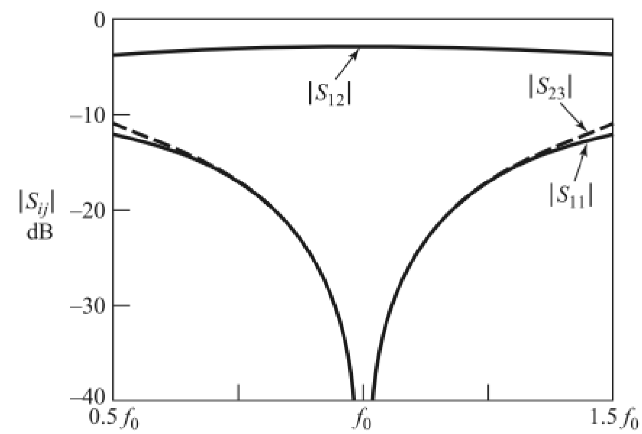
Solution
From Figure 7.8 and the above derivation, we have that the quarter-wave transmission lines in the divider should have a characteristic impedance of

$$Z = \sqrt{2} Z_0 = 70.7\ \Omega,$$

and the shunt resistor a value of

$$R = 2Z_0 = 100\ \Omega.$$

The transmission lines are $\lambda/4$ long at the frequency f_0. Using a computer-aided design tool for the analysis of microwave circuits, the scattering parameter magnitudes were calculated and plotted in Figure 7.12. ■



- For more details, refer to:
  - Chapter 7, Microwave Engineering, David Pozar_4ed.
- The lecture is available online at:
  - *http://bu.edu.eg/staff/ahmad.elbanna-courses/11983*
- For inquires, send to:
  - ahmad.elbanna@feng.bu.edu.eg